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LETTER TO THE EDITOR

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Abstract. It is shown that the holons of Zou and Anderson obey Fermi rather than Bose statistics and therefore that superconductivity *cannot* be associated with the Bose condensation of these entities. It also is shown that the mean-field theory of Kotliar and Lui is not valid for a degeneracy $N = 2$ in one dimension.

As a model for high-temperature superconductivity in the perovskites, Zou and Anderson (1988) have used the slave-boson method of Barnes (1976) to formulate the resonant-valence-bond (RVB) theory for the nearly half-filled Hubbard Hamiltonian. Because of their commutation rules, the bosons, i.e., the holons, introduced with the slave-boson method are assumed to undergo Bose–Einstein condensation. The purpose of this Letter is to show that, with the type of mean-field approximation used by Zou and Anderson, the statistics of the holons is essentially that of spinless, charged *fermions*. To obtain superconductivity involving the holons it will be necessary to cause pairs to form.

In order to illustrate the central point, it is useful to consider a simple one-dimensional tight-binding model for spinless electrons. This is a trivial model for which the result is known. The Hamiltonian is

$$\mathscr{H} = t \sum_n c^\dagger_{n+1} c_n + \text{HC}. \tag{1}$$

The slave-boson representation can be introduced in the usual fashion. There is a fermion $f^\dagger_n$ associated with the state $|n1\rangle$ which has an electron at the site n and a boson $b^\dagger_n$ associated with the empty $|n0\rangle$ site. An arbitrary operator $\hat{O}$ is replaced by its slave-boson equivalent

$$f^\dagger_n \langle n1|\hat{O}|n1\rangle f_n + f^\dagger_n \langle n1|\hat{O}|n0\rangle b_n + b^\dagger_n \langle n0|\hat{O}|n1\rangle f_n + b^\dagger_n \langle n0|\hat{O}|n0\rangle b_n$$

where, also as usual, there is a constraint

$$Q_n = f^\dagger_n f_n + b^\dagger_n b_n = 1. \tag{2}$$

The slave-boson version of the Hamiltonian is

$$\mathscr{H} = t \sum_n f^\dagger_{n+1} b_{n+1} b^\dagger_n f_n + \text{HC} \tag{3}$$

which, as will be seen explicitly below is, apart from the absence of a spin sum, identical

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to the $\mathcal{H}_0$ of Zou and Anderson (when their amplitudes $d^\dagger$ and d associated with the doubly occupied site are eliminated). Since these are spinless electrons, the spinons and their associated degrees of freedom are absent. The role the holons play in this tight-binding model is therefore surprisingly similar to their role in the infinite-U Hubbard model.

The single-particle Green function

$$G_{n,m}(\tau) = \langle T_\tau c_n^\dagger(\tau) c_m(0)\rangle \rightarrow \langle T_\tau f_n^\dagger(\tau) b_n(\tau) b_m^\dagger(0) f_m(0)\rangle. \tag{4}$$

At first sight the application of the equation-of-motion method to the *slave-boson* version of the Green function would appear intractable since it generates a set of Green functions of higher and higher order (in the sense of being n-particle). However, upon examination, it is easily seen that the constraint $Q_n = 1$ can be factored out of these higher-order Green functions reducing them all to first order. The details are of only technical interest and will not be given here since the result is trivial to obtain directly, i.e.

$$(\mathrm{i}\omega_n - \mu) G_{n,n} = 1 + t(G_{n+1,n} + G_{n-1,n}) \tag{5}$$

which can be solved in the usual fashion to give a dispersion relation $\varepsilon_k = 2t\cos k$ ($\pi k < \pi$).

It is perhaps tempting to make a mean-field approximation by replacing the Bose operators by c-numbers. This would correspond to the approach of Kotliar and Lui (1988) for the Hubbard model. On doing this, the Hamiltonian reduces to

$$\mathcal{H} = t\delta \sum_n f_{n+1}^\dagger f_n + \mathrm{HC} \tag{6}$$

which is just another tight-binding model in which the band width has been reduced by δ, the density of holes. Clearly this gives a totally wrong answer for the kinetic energy in the case of a nearly filled band. It corresponds to the case when $N = 1$, $q = 1/N$, in their notation, and shows the expansion about the large-N limit is useless for this extreme case of $N = 1$. Clearly a strategy of replacing the axillary fermion combination $f^\dagger f$ by c-numbers suffers from equivalent problems.

Such a simple mean-field approach does not work because the holon and its fermion partner are highly correlated. To illustrate this it is useful to write the Hamiltonian as

$$\mathcal{H} = t \sum_n b_n^\dagger f_n f_{n+1}^\dagger b_{n+1} + \mathrm{HC}. \tag{7}$$

The physical sub-space has only a single axillary particle for each site; thus if b_{n+1} has a finite effect, the state upon which $f_{n+1}^\dagger$ operates is the vacuum and the result is a state with a single f_n fermion. Similarly, if $b_n^\dagger$ is to have a finite effect it must be that f_n created the vacuum. To construct an accurate hole picture, it is observed that a state in the physical space is specified once the position of all the holons is given and that the interaction $\mathcal{H}$ is only effective if the site adjacent to a given holon does *not* contain another holon. In this case the combination of operators $f_n f_{n+1}^\dagger$ is a projection operator, i.e., the relevant matrix element is unity, and so in effect $f_n f_{n+1}^\dagger = 1$. It would appear that the effective Hamiltonian would be $\mathcal{H} = t\sum_n b_n^\dagger b_{n+1} + \mathrm{HC}$. However, the wavefunction constructed with the bs is even under the interchange of particles while the exact wavefunction is odd. It follows, in order to have the correct symmetry properties, that one must change the bs to fermions, so that the accurate effective Hamiltonian for holons is

$$\mathcal{H} = t \sum_n F_n^\dagger F_{n+1} + \text{HC} \tag{8}$$

where the Fs obey fermion statistics. Clearly the Fermi statistics for the holons correctly reflect the constraint on the holon occupation number, and imply that no two k-values are identical. For M holons and $\mathcal{N}$ sites, Hilbert space is spanned by the $\mathcal{N}!/(\mathcal{N} - M)!M!$ axillary real-space vectors and it is easy to check that there are the same number of k-space vectors; it follows that the solution is complete. It is also possible to create an equivalent-particle version of the theory by making the replacement $b_n^\dagger b_{n+1} = 1$.

In order to calculate the Green function in the holon theory, it is only necessary to observe that, by essentially the same argument, the individual fs in, e.g., $c_n^\dagger = f_n^\dagger b_n$ might be replaced by unity and therefore that

$$G_{n,m}(\tau) = \langle T_\tau F_n(\tau) F_m^\dagger(0) \rangle \tag{9}$$

where the replacement of the bosons with fermions leads to the appropriate periodicity in imaginary time.

Turning finally to the real problem of interest, the Hubbard Hamiltonian is

$$\mathcal{H} = t \sum_{n\sigma} (c_{n+1\sigma}^\dagger c_{n\sigma} + \text{HC}) = U \sum_n n_{n\uparrow} n_{n\downarrow} \tag{10}$$

but, in the limit of $U \to \infty$, the slave-boson version differs from the tight-binding model only in having a spin sum, i.e.,

$$\mathcal{H} = t \sum_n b_n^\dagger \left(\sum_\sigma f_n f_{n+1}^\dagger \right) b_{n+1} + \text{HC} \tag{11}$$

and $Q_n = \Sigma_\sigma f_{n\sigma}^\dagger f_{n\sigma} + b_n^\dagger b_n = 1$.

The one-dimensional problem is trivial to solve and is sufficient to illustrate that holons remain fermion particles. A treatment of the two-dimensional problem, to be described elsewhere, leads to essentially the same conclusion. The same trick can be used, i.e. it is observed that the real-space configuration of the system and matrix elements of $\mathcal{H}$ are determined once the location of all the holons is given. This works *only* in one dimension because the original electrons cannot interchange their positions. Although it is not possible to specify which combination $ff^\dagger$ is equal to unity and which is zero, for a given finite matrix element of $\mathcal{H}$, it is always the case that

$$\left\langle \left| \sum_\sigma f_{n\sigma} f_{n+1\sigma}^\dagger \right| \right\rangle = 1 \tag{12}$$

and, of course, it also remains the case that there cannot be two holons at a given lattice site. It follows that this problem reduces in exactly the same fashion as does the tight-binding model, i.e., the effective-holon Hamiltonian is simply equation (8). The only new feature is the very large spin degeneracy $2^{(\mathcal{N}-M)}$ of each holon state. It remains the case that the *holons obey Fermi statistics.* (There is *not* an equivalent-particle version of the theory in this case.)

These results for the Hubbard Hamiltonian demonstrate that the Kotliar and Lui mean-field approach also fails for $N = 2$ with $q = 1/2N$, at least in one dimension. Since there is no specific dimensional criterion involved in the existence of the large-N saddle point there is no reason to believe that it works in other spatial dimensions. It might be noted that the saddle point corresponds to a time-dependent equation for $b(\tau)$ while a static solution is assumed. It might be argued that in thermal equilibrium a stationary solution must apply; however, b is not a physical quantity and a time-dependent solution for $b(\tau)$ in which the physical expectation values are stationary cannot be excluded.

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